

Mathematics in Industry

Study Group

South Africa

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Executive Summaries

EXECUTIVE SUMMARIES

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The Executive Summary consists of a brief description of the problem followed by a largely equation-free summary of the progress made and the results obtained by the study group.

DIFFUSER OPTIMUM IMBIBITION

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Executive Summary

Sugar cane typically consists of 70% water, 15% fibre and 15% dissolved solids (mainly sugar). Optimum sucrose extraction is essential for the sugar industry. The sugar-milling process begins with the shredding of sugar cane which reduces the cane to short fibres. The shredded sugar-bearing cane is then passed through a diffuser which is a device consisting of 12-14 stages. Extraction of sugar-containing juice from the cane is facilitated by the addition of water by way of sprays positioned at intervals on the diffuser. Additional water, known as imbibition, is introduced in the final stage of the diffuser in order to maximise the extraction process. Water that was added has to be evaporated before sugar is crystallised. This evaporation process requires energy by way of burning fuel in a boiler. Much of this fuel comes from the spent shredded cane. A drawback of imbibition is that the evaporation of this additional water increases the fuel consumption and consequently the production cost, as coal needs to be bought when all the fibre has been used up. Thus, in order to reduce cost, a possible solution would be to reduce imbibition. However, would a reduction of imbibition result in lower extraction efficiencies? This is the question we explored in our work. We obtained a system of equations which allowed us to investigate this from a perspective of flow rates. We formulated a simple discrete model utilising a 12-stage diffuser. We tracked the concentration of sugar in the cane and in the juice as it made its way through the different stages. In our work, we related the horizontal and vertical flow rates in the diffuser. In particular, we focused on the vertical velocity of the liquid in the diffuser and were able to establish that a lower vertical velocity (and thus a lower vertical flow rate) produces a higher concentration of sugar in the juice. It has been well established that higher imbibition results in higher flow rates and lower concentrations in the juice, and our findings are in agreement. For future work, the operational aspects of the diffuser, for example, the idea of recycle, may be explored as means of mitigating the consequences of reduced imbibition.

TOURISM SECTOR RECOVERY PLAN

AIRLINES

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Executive Summary

The 1996 White Paper [1] on The Development and Promotion of Tourism in South Africa provides for the promotion of domestic and international tourism. The National Development Plan identifies the tourism sector as labour intensive with the potential to stimulate economic growth and transformation. It is the second largest economic sector in South Africa after the mining industry.

According to Saayman [2], a key aspect of tourism is the movement of people from one place to another. The need or desire to travel takes place for various reasons which includes, but not limited to, travelling for business or leisure, or to visit family, friends, and relatives.

Transport is used to effect the movement and therefore the transport industry makes a vital contribution to the total tourism experience. Transport in tourism has three components, namely:

- travel to the tourism destination;
- travel at the tourism destination; and
- travel from the tourism destination back to the place of residence.

Tourism transport is further divided into four types: air, rail, road, and water.

Airlines have suffered loss of revenue due to the ongoing COVID-19 pandemic and the associated travel restrictions imposed by nations to curb the spread of the virus. As part of the COVID-19 Tourism Sector Recovery Plan, aimed to reignite the tourism sector, optimisation of profit through reduction in tax on available seats has been suggested. The questions posed by the industrial representative were:

1. What changes need to be implemented?
2. How many seats need to be sold and the cost per seat taking note that due to COVID-19 regulations airlines are not permitted to carry full capacity?
3. How can airlines find ways to increase their profit using the available seats under the given COVID-19 regulations and taking into account the new Omicron variant?

The Study Group considered the problem of revenue optimisation based on ticket sales. Tickets were divided into different classes based on certain considerations (e.g. was the ticket purchaser an adult, a child, or an infant), discounts (e.g. children and infants get discounted prices) and penalties (e.g. tickets with flexible booking dates cost more). Then a revenue function was constructed as a linear combination based on the different classes of tickets, which was then maximised. The Study Group also factored in discounts based on a ticket purchasers vaccination status and introduced accompanying suitable constraints. The maximisation problem was solved using dynamic programming. Our results showed that airlines could consider approaches where a majority of tickets sold were reserved for vaccinated passengers without a significant difference or loss in the total revenue, based on actual ticket data provided by the industrial representative.

Our model did not consider any advanced or dynamic constraints, such as dynamic pricing, ticket cancellations or ticket demand. This could be considered and incorporated into future work on this problem.

References

- [1] Department of Environmental Affairs and Tourism, 1996. The Development and Promotion of Tourism in South Africa. White Paper. Available at <https://www.tourism.gov.za/aboutNDT/Publications/Tourism%20White%20Paper.pdf> (Accessed: 14 March 2022).
- [2] Saayman, M., 2013. En route with tourism. Juta, Limited.

SCALING UP FROM EXPERIMENT TO INDUSTRY

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Executive Summary

The goal of the study group was to find an explanation for the lack of agreement between results for the breakthrough curve obtained from laboratory studies, using an adsorbent in powder form, and large scale experiments, using adsorbent pellets. Specifically, the goal was to explain the qualitative difference in the breakthrough curves.

A hierarchy of models was studied during the week: First a standard model, coupling the advection-diffusion equation to a linear Langmuir model was tested. The effect of pellet size only entered through the void fraction in the column. It was demonstrated that this had a negligible effect on the form of breakthrough although it did affect the time taken.

Second, the group replaced the linear with the full Langmuir model, that is, including adsorption and desorption. Again the void fraction only appeared in the length-scale and had no effect on the solution form. Finally, following the work of [1] a third equation was introduced to the system. This dealt with diffusion within the pellets, thus introducing a new time-scale. Due to the complexity of the system it was not possible to find a numerical solution during the week. However, the intra- particle diffusion time-scale indicates that this mechanism could lead to the kind of behaviour and time-scales observed in the large-scale experiments.

Following on from the study group, it is intended to investigate the final model in more detail. It would also help if Dr Cabrera could carry out further experiments, using the same components but with different size columns.

References

- [1] Mondal R, Mondal S, Kurada K V, Bhattacharjee S, Sengupta S, Mondal M. Karmakar S, De S and Griffiths I M. Modelling the transport and adsorption dynamics of arsenic in a soil bed filter, *Chemical Engineering Science*, 210, (2019), 115205.

MOVEMENT OF BUBBLES IN A TUBE FOR METHANE EXTRACTION IN LAKE KIVU

Industry Representative:

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Executive Summary

Lake Kivu is a fresh water lake on the border between Rwanda and the Democratic Republic of the Congo. It is a deep lake with maximum depth 474 m and average depth 220 m. It contains dissolved methane and carbon dioxide in its deep waters. The amount of methane contained at the bottom of the Lake is estimated to be 65 cubic kilometres and the amount of carbon dioxide is estimated at 256 cubic kilometres. Around the Lake geologists have found evidence of massive local extinctions about every thousand years, presumably caused by outgassing events.

The government of Rwanda is reducing the amount of methane gas in the Lake by converting it to electricity. It currently has 225 MW of installed generation capacity giving 53% of the country access to electricity. It plans to give 100% of the country access to electricity by 2024.

Methane is extracted from the resource zone which is a depth between 260 m and 470 m where most of the methane is concentrated. A tube of length 320 m and diameter 1.3 m is injected into the resource zone. Bubbles are formed and their buoyancy drives a self-sustained upward flow in the tube. The Study Group was asked to investigate the bubble formation, bubble growth and movement of the bubbles within the tube. What determines where bubbles emerge initially when the temperature is raised to the point where the water is saturated with dissolved gases?

What determines where bubbles will first form? Will bubbles form near the glass, along the plastic or randomly along the length of the tube? What determines the bubble size?

The Study Group found it difficult to make progress without further information. It needed to know the nature of the flow in the tube. Is it bubbly flow, slug flow, annular flow, flow with droplets or vapour forced convection. The type of flow depends on the input conditions and the conditions at the top end of the tube. A simple slug flow model was proposed which avoids bubble dynamics and viscous drag in which Henry's law for gas release and buoyancy uplift are the primary driving mechanisms. A paper on a new model for disturbance waves in annular flow was brought to the attention of the Study Group [1]. If the water in the tube is confined to a thin annulus moving up the walls the paper will be important.

In May the Study Group established contact with Mr Tony de la Motte, Project Director, Shema Power Lake Kivu, who clarified much of the methane extraction process. Shema Power expects to start gas extraction in December 2022 and he then hopes to be able to give some real world data to assist in the problem.

References

- [1] Hall Taylor NS, Hewitt IJ, Ockendon JR and Witelski TP. A new model for disturbance waves, *Inter. J. Multiphase Flow*, **66**, 2014, 38-45.

THE DROP SHUNTING PROBLEM

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Executive Summary

A beverage company has truck and trailer combinations. It has more trailers than trucks. A trailer is loaded at the depot with the bottles of beverage for a number of clients. A truck then takes it to the first drop off point and leaves it there for that client's stock to be offloaded and the empty bottles to be loaded. The truck moves to another trailer that has completed loading and off loading at another client and moves the trailer to its next client or takes an empty trailer back to the depot or collects another loaded trailer from the depot and takes it to a client. There is therefore a pool of trucks shunting a pool of trailers around between clients and the depot. There is also a time component associated with the drop shunting problem because each trailer has different minimum times spent at each point before the next truck can come and take it to its destination. This minimum time would be roughly proportional to the size of the loading/unloading. Trailers cannot stand overnight at a client and therefore must be brought back to the depot by close of day.

The problem posed to the Study Group is to find the most efficient routing system to minimise the transportation cost in the delivery of goods. The questions are:

How do we do this while travelling the least distance?

What is the right number of trucks and trailers?

How should the deliveries be structured to minimise total cost?

The study group assumed that the Drop Shunting Problem (DSP) is either a Modification of the Vehicle Routing Problem (MVRP) that includes pick up and delivery or that it is a Job Scheduling Problem (JSP). The simple MVRP used in this study

did not capture every detail of the problem. Hence the introduction of the new formulation of the JSP that takes into account the max span of the job combination.

The study group proposed a Mixed-Integer Linear and Dynamic Programming Model with K -means clustering for solving the DSP. The K -means clustering algorithm is an interactive algorithm that tries to partition the data set in K predefined distinct non-overlapping subgroups or clusters where each data point belongs to only one group.

The developed dynamic programming method integrated with K -mean clustering proved useful for solving the DSP. Tests on a small artificial data set were used to validate the model. The results have shown that the proposed model and approach works and are practical for use in industry. A shortcoming of the work was the absence of the availability of real data sets which would provide better guidance as to what further work is required for this model.